

Analytical Mechanics

&

Differential Geometry

in honour of the 70th birthday of [Sergio Benenti](#)

March 12th and 13th 2015

On March 8, 2015 Professor Sergio Benenti will celebrate his 70th birthday. On this occasion a workshop and a special issue on SIGMA are organized to invite his friends, co-workers and colleagues from different centers around the world to contribute to a celebration of his anniversary and distinguished career achievements.

Location

[Dept. of Mathematics University of Torino](#)

Via Carlo alberto,10 - 10123 TORINO

Classroom: Aula C + Aula A (Friday afternoon)

[How to reach us](#)

Program

Thursday 12th

14.00 - 14.50 **Franco Magri** (Università di Milano Bicocca, Italy)

The Kowalevski's top 130 years later

14.50 - 15.40 **Vladimir Matveev** (University of Jena, Germany)

What is analogue of Benenti systems in the Kähler situation? C-projectivity

ABSTRACT: I will first recall what are Benenti systems, suggest different reformulations, discuss why non of them works for Kähler metrics and suggest an analogue for Kähler metrics. I show an almost algorithmic way to translate results from the classical theory of Benenti systems to the Kähler situation, and suggest you to try.

Coffee Break

16.10 - 17.00 **Stefan Rauch-Wojciechowski** (Department of Mathematics, Linköping University, Sweden)

Dynamics of rolling and sliding axially symmetric rigid bodies: Jellett's egg (JE), Tippe top rolling and sliding disc (sRD). Asymptotic solutions and numerical sampling.

ABSTRACT: Equations of motion for purely rolling axially symmetric rigid bodies have 4 degrees of freedom and are completely integrable (Routh, Chaplygin). When sliding is allowed there are 2 additional degrees of freedom for motion of centre of mass, equations are dissipative and integrability is lost. Analysis of this strongly nonlinear dynamical systems is difficult and progress is limited. The main tools in study of global dynamics is the monotonously decreasing energy function, theorems on stability of asymptotic solutions and the use of LaSallé type theorems. An additional useful property is that the underlying purely rolling problem is integrable and this can be used for studying dynamics with sliding as in the case of the Tippe top where it helped to explained oscillatory behaviour of inverting solutions of TT Equations for the Jellett's egg (an axially symmetric ellipsoid with half axis a, b) are interesting as the parameters a, b may be deformed to derive equations of other rigid bodies such as the Tippe top and the rolling and sliding Disc (sRD). This is used for understanding the common structure of all these equations and enables study of how features of dynamics change when the parameters are deformed. The use of the JE equations makes possible a uniform analysis of bifurcation diagrams of asymptotic solutions for JE, TT, sRD and are a basis for understanding of what asymptotically happens to these rolling and sliding bodies. The asymptotic solutions provide also a useful framework for numerical sampling of solutions to get an idea of what happens at different initial condition regimes. I will present numerical simulations of

inverting solutions of the tippe top. They display new interesting features; initiation of inversion requires reaching a threshold value of the angular velocity and synchronisation of remaining variables so that it is possible to discern a dynamically distinguished starting and ending time for the inversion. I shall also discuss simulations of a rolling and sliding disc. It appears that solutions above an energy threshold go asymptotically to one of the tumbling solutions (with the disc is rolling around a fixed centre of mass) but in certain regions of initial conditions the outcome is extremely sensitive to small changes of initial data.

17.00 - 17.50 **Sergio Benenti** (Università di Torino, Italy)

TBA

Friday 13th

9.00 -9.30 **Orlando Ragnisco** (Università di Roma Tre)

Classical (and Quantum) maximally superintegrable systems on spaces of non constant curvature: spectrum generating algebra and solution to the equations of motion.

ABSTRACT: The formalism of SUSYQM ({SuperSYmmetric Quantum Mechanics}) is properly modified in such a way to be suitable for the description and the solution of a classical maximally superintegrable Hamiltonian System, the so-called Taub-Nut system, depending on a parameter η .

In full agreement with the results recently derived by A. Ballesteros et al. for the quantum case, we show that the classical Taub-Nut system shares a number of essential features with the Kepler system, that is just its Euclidean version arising in the limit $\eta \rightarrow 0$, and for which a "SUSYQM" approach has been recently introduced by S. Kuru and J. Negro.

In particular, for positive η and negative energy the motion is always periodic; it turns out that the period depends upon η and goes to the Euclidean value as $\eta \rightarrow 0$. Moreover, the maximal superintegrability is preserved by the η -deformation, due to the existence of a larger symmetry group related to an η -deformed Runge-Lenz vector, which ensures that in R^3 closed orbits are again ellipses. In this context, a deformed version of the third Kepler law is also recovered.

The closing section is devoted to a discussion of the $\eta < 0$ case, where new and partly unexpected features arise.

9.30 - 10.20 **Konrad Schoebel** (University of Jena, Germany)

An operad structure on separation coordinates

ABSTRACT: We reveal a rich algebraic geometric structure behind Kalnins & Miller's famous classification of separation coordinates

on spheres. Namely, we show that the space of separation coordinates on the n -dimensional sphere modulo isometries is naturally parametrised by the moduli space of stable genus zero curves with $n+2$ marked points. As a consequence we reinterpret Kalnins & Miller's construction of separation coordinates in terms of the natural operad structure on these moduli spaces. (This is a joint work with Alexander P. Veselov.)

Coffee Break

10.50 - 11.40 **Giorgio Tondo** (Università di Trieste, Italy)

The Haantjes manifolds of Benenti systems

ABSTRACT: I will propose the new concept of symplectic-Haantjes manifolds as a general setting for analyzing integrable and separable Hamiltonian systems. As examples, I will present the symplectic-Haantjes manifolds of Staeckel and of Benenti systems.

11.40 - 12.30 **Willard Miller Jr.** (University of Minnesota, USA)

Separation of variables, superintegrability and Bôcher contractions

ABSTRACT: Quantum superintegrable systems are exactly solvable quantum eigenvalue problems. Their solvability is due to symmetry, but the symmetry is often “hidden”. The symmetry generators of 2nd order superintegrable systems in 2 dimensions close under commutation to define quadratic algebras, a generalization of Lie algebras. The irreducible representations of these algebras yields important information about the eigenvalues and eigenspaces of the quantum systems. Distinct superintegrable systems and their quadratic algebras are related by geometric contractions, induced by generalized Inönü-Wigner Lie algebra contractions which have important physical and geometric implications, such as the Askey scheme for obtaining all hypergeometric orthogonal polynomials as limits of Racah/Wilson polynomials. This can all be unified by ideas first introduced in the 1894 thesis of Bôcher to study R- separable solutions of the wave equation.

Lunch Break

14.00 - 14.50 **Franco Cardin** (Università di Padova, Italy)

Global solutions of H-J equations

ABSTRACT: Recent advances on the global --viscosity and variational-- solutions for Hamilton-Jacobi equations and the allied weak KAM theory are considered. They provide a better understanding of the trend to equilibrium of free-vorticity weak solutions for PDE systems governing inviscid Burgers fluids on tori.

14.50 - 15.40 **Pavel Winternitz** (CRM, Université de Montréal,

Canada)

General N-th order integrals of the motion and their quantization

ABSTRACT: The general form of an integral of motion that is a polynomial of order N in the momenta is presented for a Hamiltonian system in two-dimensional Euclidean space. The classical and the quantum cases are treated separately, emphasizing both the similarities and the differences between the two. The main application will be to study N th order superintegrable systems that allow separation of variables in the Hamilton-Jacobi and Schroedinger equations, respectively. This is joint work with Sarah Post.

Proceedings

A special issue dedicated to the workshop themes will be published in [SIGMA](#)

Contributions on the topics of the special issue are welcome, also from researchers who are not able to attend the workshop.

Registration

Participation is free. However, participants are invited to send an e-mail to the Organisers before March 1st.

Unfortunately no funds are available to support participation.

Accommodation

Participants are invited to make hotel reservation by their own.

Some suggested Hotel not too far from the Department are:

[Hotel Amadeus](#) *** via Principe Amedeo 41 bis (600m from the Department)

[Hotel Dock Milano](#) *** via Cernaia 46 (in front of Porta Susa railway station at 1,6km from the department)

For additional information, please contact the Organizers.

Sponsors

The workshop is supported by the Department of Mathematics "Giuseppe Peano" of the University of Torino.

Organizers

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